Exercise 25

Find the derivative of the function using the definition of derivative. State the domain of the function and the domain of its derivative.

$$f(x) = x^2 - 2x^3$$

Solution

Calculate the derivative of $f(x)$ using the definition.

$$\begin{aligned}
f'(x) &= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h\to 0} \frac{[(x+h)^2 - 2(x+h)^3] - (x^2 - 2x^3)}{h} \\
&= \lim_{h\to 0} \frac{[(x^2 + 2xh + h^2) - 2(x^3 + 3x^2h + 3xh^2 + h^3)] - x^2 + 2x^3}{h} \\
&= \lim_{h\to 0} \frac{(x^2 + 2xh + h^2 - 2x^3 - 6x^2h - 6xh^2 - 2h^3) - x^2 + 2x^3}{h} \\
&= \lim_{h\to 0} \frac{2xh + h^2 - 6x^2h - 6xh^2 - 2h^3}{h} \\
&= \lim_{h\to 0} (2x + h - 6x^2 - 6xh - 2h^2) \\
&= 2x - 6x^2
\end{aligned}$$

The domain of $f(x)$ is $\{x \mid -\infty < x < \infty\}$, and the domain of $f'(x)$ is $\{x \mid -\infty < x < \infty\}$. $f(x)$ and $f'(x)$ are polynomials, so any number can be plugged into them.